Optimization of artificial flocks by means of anisotropy measurements

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In collaboration with Motohiro Makiguchi
Aim of this study

We would like to reveal **mechanism of flocking** by constructing artificial flock in computer by taking into account **empirical evidence**

Especially, we focus on **interactions** between agents (birds) and resulting **spatio-temporal patterns** of flocks

We estimate the interactions by means of **optimization of anisotropy measurements**
An example of flocks

*Sturnus vulgaris*

- Body length: 20cm
- Wing span: 40cm
Empirical data analysis

Emergence of anisotropy (symmetry breaking)

(Ballerini et al, PNAS, 2008)

$$\phi:\text{latitude}$$

$$\alpha:\text{longitude}$$

Apparently, angular distribution is biased

Absence of birds along the direction of flock’s motion (in front & rear)
Quantifying the anisotropy

Strength of anisotropy can be evaluated by “\( \gamma \)-value”

(Ballerini et al, 2008)

\( 0 \leq \gamma < \frac{1}{3} \): anti-anisotropic

\( \gamma = \frac{1}{3} \): Isotropic (no bias)

\( \frac{1}{3} < \gamma \leq 1 \): Anisotropic
**Definition of \( \gamma \)-value**

\[
|u_i^{(n)}\rangle \equiv u_i^{(n)} = \frac{(V_{i,j^{(n:i)}} - V_i)}{|V_{i,j^{(n:i)}} - V_i|}
\]

The n-th nearest neighbouring of mate \( i \)

\[
j(n : i) \equiv \arg\min_{j \neq j(0:i), \ldots, j(n-1:i)} r_{ij}
\]

3 x 3 projection matrix

\[
M^{(n)} = \frac{1}{N} \sum_{i=1}^{N} (|u_i^{(n)}\rangle \langle u_i^{(n)}|)
\]

\[
\gamma_t = |\langle W^{(n)} | V \rangle|^2
\]

\[
\gamma = \mathbb{E}_t[\gamma_t] \equiv \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \gamma_t
\]

Eigenvector of minimum eigenvalue

\[
|V\rangle \equiv \left( \frac{1}{N} \right) \sum_{i=1}^{N} V_i
\]

Direction of flock (center of mass)
The $\gamma$-value of isotropic configuration

$$\rho(\phi, \alpha) = (4\pi)^{-1} = 1/\text{(Total solid angle)}$$

$$|\langle W^{(n)} | V \rangle|^2 = \cos^2 \phi \cos^2 \alpha$$

$$\gamma_{\text{uniform}} = \int_{\text{sphere}} \rho(\phi, \alpha) \, d\phi \, d\alpha |\langle W^{(n)} | V \rangle|^2 = \frac{1}{4\pi} \int_{-\pi}^{\pi} \cos^2 \alpha \, d\alpha \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3 \phi \, d\phi$$

$$= 1/3$$

Strength of anisotropy is measured by the gap from this value
Empirical evidence

Interactions between agents and flocking algorithm

BOIDS (Reynolds, 1987)

Agent-based modeling using only three kinds of interactions

Three interactions:

**Cohesion (C):** Making each agent’s position toward the average position of neighbouring flock mates

**Alignment (A):** Keeping the velocity of each agent the average value of neighbouring flock mates

**Separation (S):** Making a vector of each agent’s position to avoid the collision with the neighbouring flock mates
Dynamics of BOIDS

\[
V_i(l + 1) = \overline{V}_l^{(i)} e_B^{(i)}(l)
\]

\[
X_i(l + 1) = X_i(l) + V_i(l + 1)
\]

\[
e_B^{(i)}(l) = \frac{J_1 v_C^{(i)}(l) + J_2 v_A^{(i)}(l) + J_3 v_S^{(i)}(l)}{|J_1 v_C^{(i)}(l) + J_2 v_A^{(i)}(l) + J_3 v_S^{(i)}(l)|} + \eta \frac{V_i(l)}{|V_i(l)|}
\]

\[
\text{Cohesion} \quad v_C^{(i)}(l) = \frac{\sum_{j=1}^{N} \Theta(R - r_{ij}) X_j(l)}{\sum_{j=1}^{N} \Theta(R - r_{ij})} - X_i(l)
\]

\[
\text{Alignment} \quad v_A^{(i)}(l) = \frac{\sum_{j=1}^{N} \Theta(R - r_{ij}) V_j(l)}{\sum_{j=1}^{N} \Theta(R - r_{ij}) V_j(l)}
\]

\[
\text{Separation} \quad v_S^{(i)}(l) = -\frac{\Theta(R_0 - r_{ij(1:i)}) (X_{j(1:i)}(l) - X_i(l))}{|\Theta(R_0 - r_{ij(1:i)}) (X_{j(1:i)}(l) - X_i(l))|}
\]

How does one choose \( \eta \)?

\( \gamma \): Radius of visual field

Ordering the velocities within the neighbor

\[
|V_i(l + 1)| = \overline{V}_l^{(i)} \equiv \frac{\sum_{j=1}^{N} \Theta(R - r_{ij}) |V_j(l)|}{\sum_{j=1}^{N} \Theta(R - r_{ij})}
\]

\( (J_1, J_2, J_3) \)
BOIDS by a single interaction

\[(J_1, J_2, J_3)\]  

\begin{table}
\begin{tabular}{|l|c|c|c|}
\hline
Behaviour & Weight vector \( J \) & \( \gamma \)-value [SD] & FC \\
\hline
Crowded & (1,0,0) & 0.332 [0.0816] & 100\% \\
Synchronized & (0,1,0) & 0.319 [0.292] & 17.93\% \\
Spread & (0,0,1) & 0.347 [0.304] & 0\% \\
\hline
\end{tabular}
\end{table}

A set of scale-lengths in our BOIDS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of agents ((N))</td>
<td>100</td>
</tr>
<tr>
<td>Body-Length ((BL))</td>
<td>0.2 [m]</td>
</tr>
<tr>
<td>Wing-Span ((WS))</td>
<td>0.4 [m]</td>
</tr>
<tr>
<td>Radius of Separation Range ((R_0))</td>
<td>1.09 [m]</td>
</tr>
<tr>
<td>Radius of Visual Field ((R))</td>
<td>3 \times R_0 [m]</td>
</tr>
<tr>
<td>Initial Speed Average ((V'))</td>
<td>10.10 [m/s]</td>
</tr>
<tr>
<td>Initial Density of the Aggregation ((\rho))</td>
<td>0.13 [m(^{-3})]</td>
</tr>
</tbody>
</table>
Ad hoc choice of interactions

$$(J_1, J_2, J_3) = (1, 5, 1.4)$$

= (Cohesion, Alignment, Separation)

$$(J_1, J_2) = (1, 5)$$

is fixed

We change $J_3$

$\gamma \sim 0.6$

# of collision = 0
Optimization: formulation

We consider an optimization problem to maximize the cost function: $\gamma$-value for the nearest neighboring mates

With the constraint $\# \text{ of collisions} = 0$, $\# \text{ of breakups} = 0$

$$E(J) = -\gamma(J) + \lambda_1 N(J) + \lambda_2 B(J)$$

$$J = (J_1, J_2, J_3)$$

$$J_{opt} = \arg\min_J \lim_{\lambda_1, \lambda_2 \to \infty} E(J)$$

Optimal solution
A relation to the Ising model

Partition function

\[ Z = \sum_{\mathbf{s}} \exp \left[ J \sum_{\langle ij \rangle} s_i s_j \right] \]

Spin-space (micro) \( \mathbf{s} \leftrightarrow (\mathbf{V}, \mathbf{X}) \) BOIDS phase space (micro)

\[ \mathbf{J} \leftrightarrow \mathbf{J} = (J_1, J_2, J_3) \]

Ferromagnetic interaction (macro) BOIDS interaction (macro)

Procedure to determine the macro variables

\[
\begin{align*}
Z &= \sum_{\mathbf{s}} (\cdots) \rightarrow F(J) \rightarrow \text{minimize} \\
\gamma &= \mathbb{E}_t [\gamma_t] \rightarrow E(J) \rightarrow \text{minimize}
\end{align*}
\]

Time average along the sample path

To carry out the average, we need tremendous computational times

We use Genetic Algorithm
Dynamics of GA

Order of strength:
Separation > Alignment > Cohesion

\((J_1, J_2, J_3) = (0.19, 0.895, 1)\)
Behaviour of the \( \gamma \)-value

\[
(J_1, J_2, J_3) = (0.19, 0.895, 1)
\]

"anti-anisotropy" = the nearest neighbouring mates are more likely to be observed in the direction of flock’s motion ≠ empirical evidence

Empirical evidence (Ballerini et al)
Microscopic structure of anti-anisotropy

2D

Locally “crystal structure”

Direction of flock’s motion

Anti-anisotropy

3D

Numerical check:

Behaviour of volume $R^3$

$R$: average distance to the $n$-th neighbouring mate →

Almost constant up to $n=6$
Two definitions of “neighbouring” in BOIDS

**Metric model**
- Locations of each agent in metric model is the same as in topological model
- Crystal structure appears in this model
- $n_C = 3$

**Topological model**
- We attempt to reconstruct BOIDS by means of the topological model
- $n_C = 6$
Modified BOIDS dynamics

\[
\begin{align*}
\mathbf{V}_i(l + 1) &= \overline{V}_l^{(i)} \mathbf{e}_B^{(i)}(l) \\
\mathbf{X}_i(l + 1) &= \mathbf{X}_i(l) + \mathbf{V}_i(l + 1)
\end{align*}
\]

\begin{align*}
\mathbf{v}_{TC}^{(i)}(l) &= \frac{\sum_{j=1}^N \Theta(R_n^{(i)} - r_{ij}) \mathbf{X}_j(l)}{|\sum_{j=1}^N \Theta(R_n^{(i)} - r_{ij}) \mathbf{X}_j(l)|} - \mathbf{X}_i(l) \\
\mathbf{v}_A^{(i)}(l) &= \frac{\sum_{j=1}^N \Theta(R_n^{(i)} - r_{ij}) \mathbf{V}_j(l)}{|\sum_{j=1}^N \Theta(R_n^{(i)} - r_{ij}) \mathbf{V}_j(l)|} \\
\mathbf{v}_{GC}^{(i)}(l) &= \frac{\sum_{j=1}^N \mathbf{X}_j(l)}{|\sum_{j=1}^N \mathbf{X}_j(l)|} - \mathbf{X}_i(l) \\
\mathbf{v}_{MS}^{(i)}(l) &= -\frac{\sum_{n=1}^{n_c} \Theta(R_n^{(i)} - r_{ij(n:i)}) (\mathbf{X}_{j(n:i)}(l) - \mathbf{X}_i(l))}{|\sum_{n=1}^{n_c} \Theta(R_n^{(i)} - r_{ij(n:i)}) (\mathbf{X}_{j(n:i)}(l) - \mathbf{X}_i(l))|}
\end{align*}

\[R_n^{(i)} = |r_0^{(i)}| n^{1/3}\]

From Cavagna (2010)

The n-th nearest neighbouring of mate i

\[j(n : i) \equiv \arg\min_{j \neq j(n-1:i), \ldots, j(0:i)} r_{ij}\]
GA dynamics

A set of scale-lengths in the topological model

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</tr>
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<td>Initial Density of the Aggregation ($\rho$)</td>
<td>0.54 [m$^{-3}$]</td>
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Cf. Metric model

\[ J_3 > J_2 > J_1 \]

\[ J_2 > J_3 > J_4 > J_1 \]
γ-value and volume to the n-th neighbouring

Anti-anisotropy vanishes

Crystal structure vanishes

They behave like “gas” rather than “crystal”

Let us check the demo
Demo
Summary

We provide a novel strategy to determine the weights of interactions --- to design artificial flocks --- in the BOIDS by maximizing the $\gamma$-value by using GA.

There are some empirical gaps between BOIDS simulation and real flock of starlings.
Other mathematical models


\[
\begin{align*}
\mathbf{x}_i(t+1) &= \mathbf{x}_i(t) + \mathbf{v}_i(t) \Delta t \\
\theta(t+1) &= \langle \theta(t) \rangle_r + \Delta \theta_{\text{noise}}
\end{align*}
\]

Average over the mates within radius \( r \) of visual field

\[
v_a = \frac{1}{N} \sum_{i=1}^{N} v_i
\]

Phase transition

Noise strength


\[
\begin{align*}
\dot{q} &= p \\
\dot{p} &= -\nabla V(q) - \hat{L}(q)p + f_\gamma(p, q, q_r, p_r)
\end{align*}
\]

Potential to form “crystal structure”  Navigation by a leader

\[\rightarrow \neq \text{empirical evidence}\]
Other physical quantities

Integrated conditional density

\[ \Gamma(r) = \frac{1}{n_c} \sum_{i=1}^{n_c} \frac{N_i(r)}{4\pi r^3/3} \]

Pair distribution function

\[ g(r) = \frac{1}{4\pi^2 n_c} \sum_{i=1}^{n_c} \sum_{j \neq i} \delta(r - r_{ij}) \]

Metric model

Topological model

Metric model

Topological model